Dan's Tutorial on Linear Elastic Material Properties

Bulk material properties measure how a material deforms when you push on it. The most commonly referenced bulk material properties capture the properties of *linear elastic* materials. Those are the ones I'm going to discuss here; the point of this tutorial is to remind myself what the various moduli are when I need to look them up.

First, what does "linear elastic" mean?

- An *elastic* material wants to resist a change in shape, so if you try to squish it, it pushes back at you. It also returns to its original shape when you stop squishing it.
- A *linear elastic* material has a linear relationship between how hard you squish it and how much it deforms. So if I squeeze a block of linear elastic material with two pounds of force, it will get squished twice as far as it would if I squeezed with one pound of force.

Do real materials that we care about actually have these properties? In general, solids like plastic and metal behave more or less as linear elastic materials unless you *really* bend or squish them. Above some threshold amount of force, materials become both nonlinear and non-elastic, at which point all of the quantities discussed in this summary become meaningless. After that point, materials will usually permanently bend and eventually crack.

Now a couple of quick definitions…

- "stress" is the engineering word for pressure, and generally has units of force/area.
- "strain" is the engineering word for deformation, and generally represents a fractional change in volume (i.e. an object with a measured strain of 0.2 has increased its volume by 20% relative to its natural rest volume).

So on to the useful material properties…

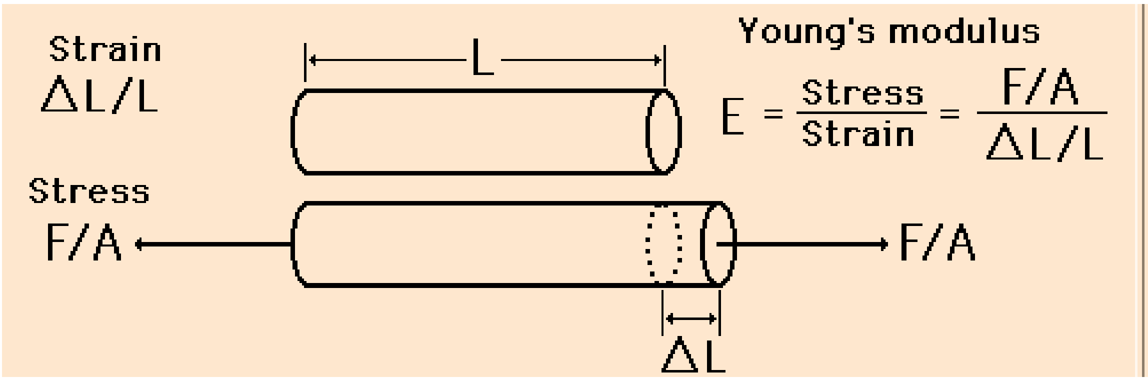
- The "bulk modulus" (**B**) represents the ratio of uniform stress to uniform strain. I.e. if I take a ball of metal and push equally into it from all sides, the bulk modulus tells me how much strain (shrinking) I'll get. It's the *inverse* of compressibility, so a higher bulk modulus means less strain per unit stress (higher **B** → harder to squish). Mathematically, **B** is usually expressed as:

  $B = P / (\Delta V/V)$

  …or the pressure required for a unit change in volume. The units of **B** are thus pressure units (e.g. pascals or N/m^2).

- The "Young's modulus" (**Y**) (or sometimes **E**) represents the ratio of uniaxial stress to uniaxial strain.

  The bulk modulus (discussed above) tells me what a material does when I push or pull it equally in all directions. But if I have a brick-shaped piece of material and I pull it only along its long axis, it will change both volume and shape (since my brick will tend to get *thinner* as I make it *longer*). So the bulk modulus won't give me a very good description of what my material does. The Young's modulus (**Y**) tells me how much a material will stretch along a single axis if I pull it along that axis… this figure that I stole from the web illustrates this:



Like the bulk modulus, **Y** is defined in terms of stress/strain. Higher **Y** → harder to stretch. The units are again pressure units (e.g. pascals or N/m^2). Numerically, the definition of Y is:

$Y = S/(\Delta L/L)$

…where S is the uniaxial stress (how hard you're pulling) and L is the length of the object.

- A material's "poisson's ratio" (**v**) (or sometimes **σ**) tells me how much a material shrinks along its cross-section when I pull it along an axis. In the above picture, the cylinder probably gets *thinner* when I pull it along its axis, since materials tend to resist volume changes. Poisson's ratio tells me *how much thinner* an object will get.

  Numerically, v is defined as:

  $v = -(\Delta W/W) / (\Delta L/L)$

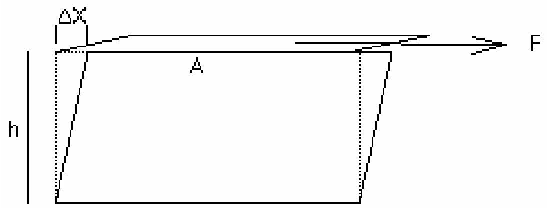
  …the change in width per unit change in length.

  This quantity does *not* measure how "stiff" an object is; there are no forces in the definition of **v**. The question is answers is "I pushed 5cm along this axis, what did this other axis do?" Maybe I had to push like hella-crazy hard to get there, maybe it was easy… the value of **v** has some intuitive interpretations:

  - What does it mean if v is :
    - 0 : completely compressible, doesn't feel that it needs to expand in one direction to accommodate change in another (e.g. sponge, cork)
    - 0.5 : completely incompressible, expands to exactly compensate for volume changes (water or steel are very close to this)
    - <0 : Actually gets *wider* if I pull on the long axis… the only example I found was an "unwinding filament" object that someone made to flex their materials science skillz.

- A material's "shear modulus" (μ) (or sometimes **G**) tells me how hard I have to push to *shear* the material. Like the other moduli, it is a ratio of stress to strain: how hard I have to push to induce a certain amount of deformation. The bulk modulus looked at uniform stresses, the Young's modulus looked at uniaxial stresses, and the shear modulus looks at *shear stresses*.

  A shear stress is a stress that's applied that tends to twist a material… this picture that I stole from the web illustrates the idea:



Here F is a force that I'm applying to slide the top of this block while the bottom of the block is fixed in place. A is the area over which I apply the force, h is the height of the material, and Δx is the amount that I manage to slide the top away from the bottom. The "shear strain" is defined as $\Delta x/h$, so you can guess what the definition of μ is going to look like:

$$\mu = (F/A) / (\Delta x/h)$$

Higher $\mu \rightarrow$ harder to shear.