A Novel Framework for Pulse Pressure Wave Analysis Using Persistent Homology

Saba Emrani, T. Scott Saponas, Dan Morris, and Hamid Krim, *Fellow, IEEE*

***Abstract*—Four characteristic points of pulse pressure waves–the systolic peak, the anacrotic notch, the dicrotic notch, and the diastolic foot–are used to estimate various aspects of cardiovascular function, such as heart rate and augmentation index. We propose a novel approach to extracting these characteristic points using a topological signal processing framework. We characterize the topology of the signals using a collection of *persistence intervals*, which are encapsulated in a *persistence diagram*. The characteristic points are identified based on their time of occurrence and their distance from the identity line in the persistence diagram. We validate this approach by collecting radial pulse pressure data from twenty-eight participants using a wearable tonometer, and computing the peripheral augmentation index using a traditional derivative-based method and our novel persistence-based method. The augmentation index values computed using the two methods are statistically indistinguishable, suggesting that this representation merits further exploration as a tool for analyzing pulse pressure waves.**

***Index Terms*—Algebraic topology algorithms, pulse pressure waves, topological signal analysis, wearable sensors.**

I. Introduction

THE arterial pulse pressure waveform is the time-varying pressure exerted by blood on the wall of an artery. A pulse pressure waveform can be measured invasively (using a pressure-sensing catheter) or non-invasively (using an external pressure sensor placed against the skin near an artery). When measured in or near the heart, the pressure waveform is referred to as a *central* pressure wave; when measured at a site far from the heart, the pressure waveform is referred to as a *peripheral* wave. In this paper, we will specifically focus on *non-invasive*, *peripheral* waveforms, specifically measured at the radial artery in the wrist. The arterial pressure waveform is a result not only of the heart beating and expelling a pressure wave to move blood forward through the arteries, but also of that pressure wave reflecting at various points in the arterial system and traveling back toward the heart. The observed waveform is the sum of these *forward* and *reflected* waves. If the reflected wave is large relative to the forward wave, the heart is expending excess energy to expel blood against the force of its own pressure wave, so we refer to the pressure seen by the heart as being "augmented" by the reflected wave. The degree to which the pressure is so "augmented" is referred to as the *augmentation index* (AIx), a function of both arterial stiffness and the geometry that leads to pulse wave reflection. AIx is known to correlate with the presence of cardiovascular risk factors such as hypertension, diabetes, and high cholesterol [1]–[4], and is a known predictor of cardiovascular mortality in patients with chronic kidney disease [5]. When measured peripherally, AIx is defined as the ratio of late systolic pressure to early systolic pressure, where early systolic pressure is the highest pressure measured at any point in the pressure wave, and late systolic pressure represents the pressure measured at the instant the reflected wave arrives at the measurement site (Fig. 1). The arrival of the reflected wave is visible as an inflection point (or sometimes a slope reversal) in the pressure wave, known as the *anacrotic notch*. However, finding the systolic peak and the anacrotic notch is often non-trivial in the presence of noise.

Measuring the augmentation index near the heart (*central AIx*) provides more robust information about cardiovascular risk than measuring at the periphery (*peripheral AIx*), but central measurement is an invasive procedure. The present work is motivated by the emergence of continuous health monitoring tools; we are interested in developing methods suitable for wearable, non-invasive devices, and the signal processing challenges that arise when measuring pressure in noisy, ambulatory contexts. Furthermore, it is shown in [6] and [7] that central AIx is closely correlated with peripheral AIx, and that information about central wave reflection can be obtained from the radial pulse. In this study, we therefore calculate peripheral AIx using the radial pressure waveform.

We propose a new framework for extracting the characteristic points of the peripheral pressure wave based on persistent homology [8], and we test this framework on real data. Specifically, using the persistence diagram, we locate key characteristic points on the pressure wave, and use these points both for AIx calculation and pulse validation. We use a wearable tonometer with three pressure sensors mounted on a wristband to measure the pulse pressure waves on a group of twenty-eight subjects. We show that the novel persistence-based analysis produces AIx values that are not statistically different from those computed using traditional derivative-based techniques.

This study introduces a new perspective on the described problem and provides researchers with an additional set of tools for developing ambulatory pulse sensors and solving noise reduction problems in pressure wave analysis.

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S. Emrani and H. Krim are with the Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC 27606 USA (e-mail: semrani@ncsu.edu; ahk@ncsu.edu).

T. S. Saponas and D. Morris are with Microsoft Research, Redmond, WA 98052 USA (e-mail: ssaponas@microsoft.com; dan@microsoft.com).

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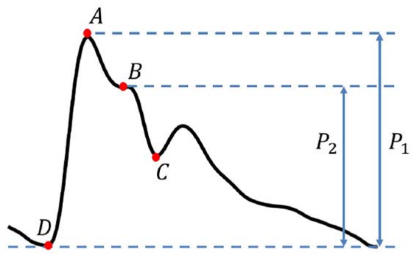

Fig. 1. Characteristic points of pulse pressure wave shown on a sample pulse acquired from the wearable tonometer. The three characteristic points we identify using persistent homology are the (a) systolic peak, (b) anacrotic notch, (c) dicrotic notch, and (d) diastolic foot.

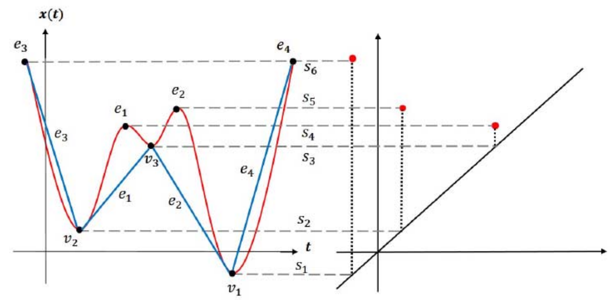

Fig. 2. Construction of a persistence diagram for a single-variable function.

The remainder of the paper is organized as follows: The proposed framework for AIx analysis and pulse validation is discussed in Section II. The data collection procedure, the characteristics of the wearable tonometer sensor, and experimental results are presented in Section III. Finally, Section IV concludes the paper.

II. Proposed Framework

Four feature points characterize the peripheral pressure waveform: the systolic peak (the point of highest pressure), the anacrotic notch (the inflection point or slope reversal corresponding to the arrival of the reflected wave), the dicrotic notch (a local minimum in pressure corresponding to closure of the aortic valve), and the diastolic foot (the point of lowest pressure). These points are represented by points A, B, C, and D respectively in Fig. 1. Peripheral AIx is defined as the ratio of late systolic pressure (P_2) to early systolic pressure (P_1) as illustrated in [9], where P_1 is the difference between the pressure at the anacrotic notch and the baseline (diastolic) pressure, and P_2 is the difference between the systolic and baseline (diastolic) pressures. Consequently, in order to compute AIx, we need to locate the anacrotic notch and the systolic peak relative to the baseline pressure. We will use the persistence diagram of the first derivative of the pulse pressure signal to obtain this information.

A. Persistence

In this subsection, we provide a brief overview of persistent homology for single variable functions, and summarize the algorithm for constructing the persistence diagram. We refer the reader to [10] for more information. Consider a continuous function $x(t) : \mathbb{R} \to \mathbb{R}$ with continuous derivatives. $t_c \in \mathbb{R}$ is a critical point of x if $x'(t_c) = 0$ and it is a non-degenerate critical point if $x''(t_c) \neq 0$. If all critical points of x are non-degenerate with distinct values, then each critical point will be a local minimum or maximum. The sublevel sets of the function x are defined as $\mathbb{R}_s = x^{-1}(-\infty, s]$ for each $s \in \mathbb{R}$. To find the persistent homology of the function, we start with $s = -\infty$ and consider the connectedness of sublevel sets as s increases. The connectivity of $\mathbb{R}_s$ does not change while increasing s, except when we pass a non-degenerate critical value. If the corresponding critical point is a local minimum, the sublevel set adds a new component; if the critical point is a local maximum, two components merge into one.

We match the critical values of x as s increases using the following procedure. When a new component appears, we consider the local minimum that established it as the component representative. As we reach a local maximum merging two components, we match it with the component representative that appeared later, which is the higher local minimum. The remaining component is then represented by the other local minimum. Accordingly, the local minima are not necessarily matched to their neighboring local maxima. When two critical values t_1 and t_2 are matched together, the persistence of the combination is defined as $x(t_2) - x(t_1)$. We map each pair (t_1, t_2) to a point $((x(t_1), x(t_2))$ and all these points are encapsulated in a structure called the *persistence diagram*. All points in the persistence diagram are above the diagonal line, and the vertical distance of each point to this line represents the *persistence* of the corresponding pair.

The information we need to form the persistence diagram includes the levels at which each connected component appears or merges with another one as well as the pairings between these events. In order to obtain this information, we utilize a filtration on a graph constructed for each function. Starting with $s = -\infty$, as the value of s in the level sets increases and the horizontal line moves upwards, we add a vertex where a connected component appears and an edge where two connected components merge. Therefore, vertices and edges will correspond to local minima and maxima, respectively. Moreover, we store the level values at which these events happen. We then form a graph where two vertices corresponding to two connected components are linked by an edge; this edge corresponds to the local maximum merging them. Using the described graph, we create a boundary matrix representing the vertices as rows and the edges as columns. Assuming an ordering on vertices and edges, the matrix will be described by $\partial_p = [a_i^j]$, where i ranges from n_v to 1, j ranges from 1 to n_e, and n_v and n_e are the number of vertices and edges, respectively. The elements of the matrix are computed as follows, $a_i^j = 1$ if and only if vertex v_i is connected to edge e_j (a_i^j is zero otherwise). We perform a column reduction on this matrix and then mark the first nonzero elements in all columns. The determined nonzero elements with the stored level values will specify the pairings. For example, if b_i^j is one of the specified nonzero elements, we conclude that the vertex v_i is paired with the edge e_j, therefore we add point (s_{v_i}, s_{e_j}) to the persistence diagram, where s_{v_i} and s_{e_j} are the levels of v_i and e_j, respectively. Fig. 2 shows an example signal with its corresponding graph built using the framework described above.

The boundary matrix of the graph and the result of performing column reduction is represented below.

$$\begin{array}{c} \\ v_3 \\ v_2 \\ v_1 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array} \rightarrow \begin{array}{c} \\ v_3 \\ v_2 \\ v_1 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & \end{array} \\ \begin{pmatrix} \mathbf{1} & 0 & 0 & 0 \\ 1 & \mathbf{1} & 0 & 0 \\ 0 & 1 & \mathbf{1} & 0 \end{pmatrix} \end{array}$$

The locations of the first nonzero elements in each column (bold 1's) determine the pairings as (v_3, e_1), (v_2, e_2), and (v_1, e_3). We can then use the levels of the vertices and edges to obtain the points in the persistence diagram as (s_3, s_4), (s_2, s_5), and (s_1, s_6) as illustrated in the right side of Fig. 2.

An important phenomenon that makes the proposed technique reliable is the stability of the persistence diagram [11], which implies that small variations in the signal imply smaller changes in the diagram. Accordingly, persistence diagrams provide a stable representation of the topological properties of pulse pressure waves.

B. AIx Calculation and Pulse Validation

In this subsection, we describe the application of persistent homology to pulse wave analysis. Data collection, preprocessing, and pulse segmentation (the separation of a data stream into individual pulses) are described in Section III. The method for computing the augmentation index (AIx) described in this subsection operates on individual pulses.

Peripheral AIx is computed as $\frac{P_2}{P_1}$ (Fig. 1); we therefore need to locate the anacrotic notch and systolic peak in each pulse and compute their distances to the baseline in order to compute AIx. The anacrotic notch is traditionally identified as an inflection point (a zero crossing of the second derivative, specifically a local maximum in the first derivative) of the pulse pressure signal [12]. The systolic peak is generally defined as the point of maximum pressure in the pulse.

In this study, we use the persistence diagram of the pulse wave and its derivative to locate characteristic points. The systolic peak is identified as the point in the persistence diagram of the pulse pressure wave that is farthest from the $y = x$ line, i.e. the local maximum with the longest lifetime. The anacrotic notch–traditionally defined as an inflection point in the pressure wave–is identified as the point in the persistence diagram of the *derivative* of the pressure wave having the third-farthest distance from the $y = x$ line (the systolic peak and dicrotic notch will be further from the $y = x$ line in persistence space). Note that besides the critical points located far from the diagonal, there are some points very close to the identity line caused by small variations in the signal. Since in this study we consider only points far from the diagonal, those points attributed to noise will not affect our results.

Pulse validation (determining whether a pulse is a physiologically valid pulse, not corrupted by motion or other artifact) is also performed using the identified times of the systolic peak and anacrotic notch, and confirming that they meet physiological and empirical constraints. For this study, this validation is performed after all pulses are processed for a recording. Specifically, a valid pulse is one in which the anacrotic notch is no more than 150 ms after the systolic peak, and the distance between the point corresponding to the anacrotic notch and the $y = x$ line in the persistence diagram is within twice the standard deviation of such distances for a whole recording. Pulses classified as "invalid" are discarded from further analysis (less than 1% of pulses in the experiments discussed below were labeled as invalid).

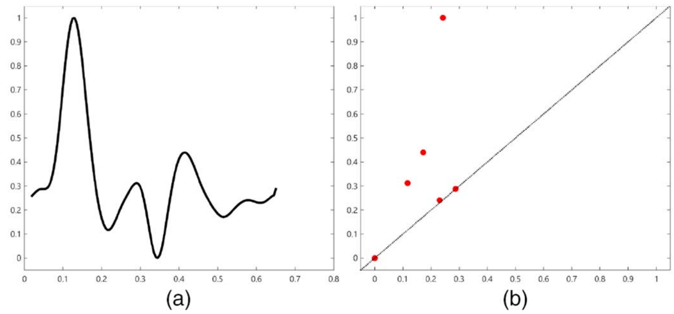

Fig. 3. The first derivative of the pulse pressure wave in Fig. 1 (left) and its persistence diagram (right).

III. Data Acquisition and Experimental Results

The signal measurements were carried out on a group of volunteers at our institution. The group consisted of 28 subjects (16 male) between the ages of 22 and 65 years (mean 33.6).[1] Data collection was conducted using a wearable tonometer mounted in a wristband. The wearable tonometer uses three MS5805 sensors in a triangular configuration (Measurement Specialties, Inc., Hampton, VA, USA) (equilateral triangle, with a center-to-center spacing of about 0.6 cm). The MS5805 is a piezoresistive sensor packaged with a 24-bit analog-to-digital converter and a protective plastic housing, providing absolute pressure measurements from 1 kPa to 200 kPa. The three sensors were interfaced with an Arduino Due microprocessor board and sampled at 204 Hz. A silicone gel was applied to the top of each sensor to fill the sensor housing. Three of these modified sensors are mounted on a custom printed circuit board (PCB). This PCB is then mounted in a custom plastic housing that attaches to a fabric wristband, which includes a Velcro fastening loop. The pulse pressure waveform was collected from the radial artery on the left wrist while the subjects were in resting position with their hands on a desk, for approximately one minute per recording. Two recordings were performed for each participant.

Because our approach computes the persistence diagram for individual pulses, pulses need to be segmented from the data stream. Each one-minute data recording was downsampled from 204 Hz to 200 Hz, then passed through a an FIR low-pass filter with a transition band between 18 Hz and 19 Hz. Heart rate was computed in 15-second windows (with 80% overlap) by computing the autocorrelogram (3-second support) and finding the median of the dominant autocorrelation frequencies. Pulses were segmented by finding local minima in the filtered data (corresponding to candidate diastolic feet), enforcing a minimum

[1]Thirty participants were recruited initially, but two were dropped prior to analysis due to difficulty in waveform acquisition.

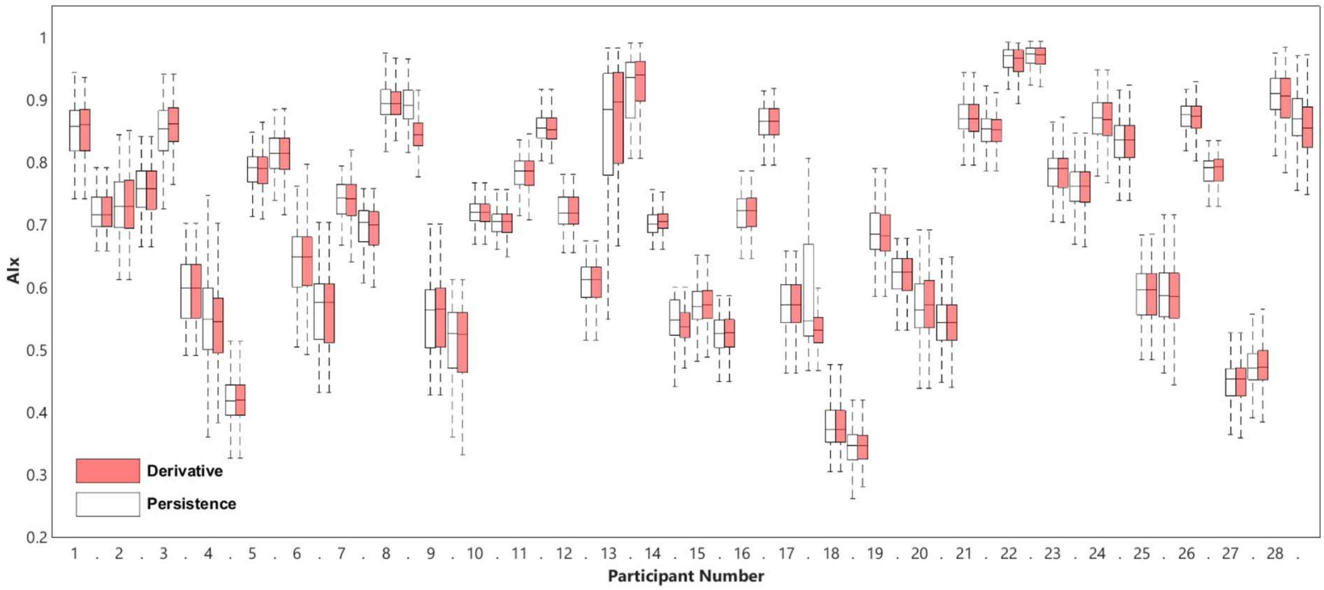

Fig. 4. AIx values calculated using the derivative method (red boxes) and the persistence technique (white boxes) in two recordings for each participant.

time between peaks equal to two-thirds of the heart rate estimate. Each span between two local minima is considered a candidate pulse.

We remove the DC bias of each pulse wave by subtracting its mean, and normalize each pulse to the range [0,1]. The first derivative of each pulse is then calculated based on a five-point scheme. The systolic peak and anacrotic notch of each pulse are identified using the persistence method described in Section II, and the AIx is computed for each pulse.

We locate the critical points and compute the augmentation index using the traditional derivative-based method [12], for comparison. In this method, the systolic peak is first identified as the largest point in the waveform, the dicrotic notch is identified as the first local minimum in the signal at least 100 ms after the systolic peak, and the anacrotic notch is identified as the first local maximum in the derivative that is no more than 150 ms after the systolic peak (the same time constraint used for the persistence method). Pulse validation uses the same heuristics: if no suitable peaks in the pressure signal or the first derivative are found meeting the criteria for the dicrotic and anacrotic notches, respectively, the pulse is classified as "invalid" and discarded from further analysis (less than 1% of pulses were labeled as invalid).

Fig. 4 illustrates the augmentation indices calculated using the derivative-based method (red boxes) and using the persistence technique (white boxes). Each box represents one recording; the central mark represents the median augmentation index computed for all pulses in that recording, and the bottom and top of the box represent the first and third quartiles. The ends of the whiskers show the most extreme AIx values not considered outliers (a point is considered an outlier if it is more than $1.5{*}(q3 - q1)$ above q3 or below q1 percentile, where q1 and q3 are the 25th and 75th percentiles, respectively. Two boxes are shown for each participant, one for each of the two recordings. Fig. 5 compare the mean of AIx values computed for each recording using the derivative and persistence methods. The Spearman correlation between these

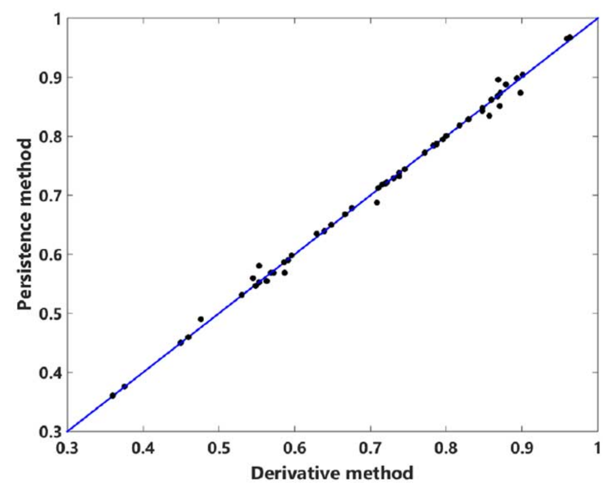

Fig. 5. The correlation between AIx values calculated using the persistence and derivative methods for each of two recordings for all participants. The solid line is $y = x$.

results is $r = 0.9973$, confirming that AIx values generated by the persistence technique are nearly identical to those computed using the traditional method.

IV. Conclusion

In this study, a novel framework for the analysis of pressure wave morphology is proposed. We calculated peripheral AIx based on topological signal processing and used the locations of the points in the persistence diagram to locate characteristic points in the pressure waves. The AIx values computed using the introduced approach on real data are statistically indistinguishable from those generated using a traditional derivative-based method. Persistence analysis thus offers a new set of tools for researchers working on pressure wave morphology analysis. Future work will focus on assessing the utility of this approach for noise reduction, and on evaluating the persistence diagram as a visualization tool for understanding temporal patterns in morphological landmarks.

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